

Bipolar Pythagorean Neutrosophic Soft Generalized Pre-Continuous Mappings

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Abstract

The aim of this paper is to introduce and investigate some new class of mappings such as bipolar pythagorean neutrosophic soft continuous mappings and bipolar pythagorean neutrosophic soft generalized pre-continuous and irresolute mappings in bipolar pythagorean neutrosophic soft topological space. Topological structures like closeness, openness, compactness etc., are preserved by continuity mappings. Different continuous mappings based on the bipolar pythagorean neutrosophic soft set is investigated. Also, some of their fundamental properties are studied and its relation with other mappings are given with examples.

Keywords: bipolar pythagorean neutrosophic soft set (BPNSS), BPNS-continuous mappings, BPNSGP-continuous mappings, BPNSGP-irresolute mappings.

1 Introduction

Zadeh [13] introduced fuzzy set in which each element is assigned a membership degree in the form of a single crisp value in the interval $[0,1]$. The generalization of fuzzy set and fuzzy logic to intuitionistic fuzzy sets (IFS) by adding the falsehood (f), the degree of non-membership was introduced by Atanassov [5] to have a better accuracy level. The definition for some operations on intuitionistic fuzzy set and its properties was given by Atanassov[6]. The concept of intuitionistic fuzzy topological spaces was put forward by Coker [7].

Smarandache[12] introduced neutrosophic set, it deals with membership, indeterminacy and non-membership degree. But it is applicable only on three attribute values. The topology ideas for neutrosophic sets were introduced by Salama et al.[11]. In 1999 soft set theory is introduced by Molodtsov[8]. Anitha et.al[4] developed NGSR closed sets. In 1970, generalied closed set and some continuity mappings in point set topology is developed by Norman Levine[9,10]. Anitha et al[1,2,3] introduced the concept bipolar pythagorean neutrosophic soft set and studied its properties.

In this paper, bipolar pythagorean neutrosophic soft generalized pre-continuous mappings are introduced. This paper is organized as follows: Section 2 consists the required preliminary definitions for main concept. Section 3 has the projected BPNS-continuous mapping and its relation with other mapping. Section 4 consists bipolar pythagorean neutrosophic soft generalized pre-continuous mappings. This section deals with theorems related to the proposed mappings and examples. In section 5, bipolar pythagorean neutrosophic soft generalized pre-irresolute mappings are introduced. Section 6 concludes the proposed work.

2. Preliminaries

Definition: 2.1 [1] Let U be a universe and E be a set of parameters. A bipolar pythagorean neutrosophic soft set (BPNSS) $\mathbb{A} = \{(e, \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)) : u \in U\}) : e \in E\}$ where $T^+ : X \rightarrow [0,1]$, $I^+ : X \rightarrow [0,1]$, $F^+ : X \rightarrow [0,1]$, $T^- : X \rightarrow [-1,0]$, $I^- : X \rightarrow [-1,0]$, $F^- : X \rightarrow [-1,0]$ are the mappings such that $0 \leq ((T^+(u))^2 + (I^+(u))^2 + (F^+(u))^2) \leq 2$ and $-2 \leq -((T^-(u))^2 + (I^-(u))^2 + (F^-(u))^2) \leq 0$. The positive membership degree $T^+(u), I^+(u), F^+(u)$ denote the Truth, Indeterminacy and False membership of an element corresponding to a bipolar pythagorean neutrosophic soft set \mathbb{A} and the negative membership degree $T^-(u), I^-(u), F^-(u)$ denotes the Truth, Indeterminacy and False membership of an element $u \in U$ to some implicit counter property corresponding to a bipolar pythagorean neutrosophic soft set.

Definition: 2.2[9] Let (X, τ) be a topological space. For any subset $Y \in X$ and $Y \subseteq U$ and U is open in X ,

- i) $cl(Y) \subseteq U$, then Y is generalized closed set (g-closed)
- ii) $scl(Y) \subseteq U$, then Y is generalized semi closed set (gs-closed)
- iii) $pcl(Y) \subseteq U$, then Y is generalized pre-closed set (gp-closed)
- iv) $spcl(Y) \subseteq U$, then Y is generalized semi pre-closed set (gsp-closed)
- v) $\alpha cl(Y) \subseteq U$, then Y is α -generalized closed set (α g-closed)

Definition: 2.3 [2] A BPNSS \mathbb{A} of a BPNS topological space X is called bipolar pythagorean neutrosophic soft generalized pre-closed set (BPNSPCS) if $BPNSpcl(\mathbb{A}) \subseteq U$ whenever $\mathbb{A} \subseteq U$ and U is BPNSOS in X .

Definition: 2.4 [9] Let (X, τ) and (Y, σ) be any two topological spaces. For every closed set V of (Y, σ) , a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

1. semi-continuous if $f^{-1}(V)$ is semi-closed set in (X, τ) .
2. pre-continuous if $f^{-1}(V)$ is pre-closed set in (X, τ) .
3. semi pre-continuous if $f^{-1}(V)$ is semi pre-closed set in (X, τ) .
4. α -continuous if $f^{-1}(V)$ is α -closed set in (X, τ) .
5. generalized-continuous if $f^{-1}(V)$ is generalized-closed set in (X, τ) .
6. generalized semi-continuous if $f^{-1}(V)$ is generalized semi-closed set in (X, τ) .
7. generalized pre-continuous if $f^{-1}(V)$ is generalized pre-closed set in (X, τ) .
8. generalized semi pre-continuous if $f^{-1}(V)$ is generalized semi pre-closed set in (X, τ) .
9. α -generalized continuous if $f^{-1}(V)$ is α -generalized closed set in (X, τ) .

3. Bipolar Pythagorean Neutrosophic Soft Continuous Mappings

Definition: 3.1 Consider two bipolar pythagorean neutrosophic soft topological spaces (U, τ, E) and (V, σ, E') . For every bipolar neutrosophic soft closed set A of (V, σ, E') , a map $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ is,

1. Bipolar pythagorean neutrosophic soft continuous (BPNS-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft closed set in (U, τ, E) .
2. Bipolar pythagorean neutrosophic soft semi-continuous (BPNSS-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft semi-closed set in (U, τ, E) .
3. Bipolar pythagorean neutrosophic soft pre-continuous (BPNSP-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft pre-closed set in (U, τ, E) .
4. Bipolar pythagorean neutrosophic soft semi pre-continuous (BPNSSP-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft semi pre-closed set in (U, τ, E) .
5. Bipolar pythagorean neutrosophic soft α -continuous (BPNS α -continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft α -closed set in (U, τ, E) .
6. Bipolar pythagorean neutrosophic soft regular continuous (BPNSR-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft regular closed set in (U, τ, E) .

Definition: 3.2 Consider two bipolar pythagorean neutrosophic soft topological spaces (U, τ, E) and (Y, σ, E') . For every bipolar neutrosophic soft closed set A of (V, σ, E') , a map $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ is,

1. Bipolar pythagorean neutrosophic soft generalized continuous (BPNSG-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized closed set in (U, τ, E) .
2. Bipolar pythagorean neutrosophic soft generalized semi-continuous (BPNSGS-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized semi-closed set in (U, τ, E) .
3. Bipolar pythagorean neutrosophic soft generalized pre-continuous (BPNSGP-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized pre-closed set in (U, τ, E) .
4. Bipolar pythagorean neutrosophic soft generalized semi pre-continuous (BPNSGSP-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized semi pre-closed set in (U, τ, E) .
5. Bipolar pythagorean neutrosophic soft generalized α -continuous (BPNSG α -continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized α -closed set in (U, τ, E) .
6. Bipolar pythagorean neutrosophic soft generalized regular continuous (BPNSGR-continuous) if $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized regular closed set in (U, τ, E) .

4 Bipolar Pythagorean Neutrosophic Soft Generalized Pre-Continuous Mappings

Definition 4.1 Suppose that $f^{-1}(A)$ is bipolar pythagorean neutrosophic soft generalized pre closed set (BPNSGPCS) in (U, τ, E) for every bipolar pythagorean neutrosophic soft closed set (BPNSCS) B of (V, σ, E') then a map $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ is said to bipolar pythagorean neutrosophic soft generalized pre-continuous (BPNSGP-continuous) mapping.

Example 4.2 Let $U = \{u_1, u_2\}$, $V = \{v_1, v_2\}$ and $E = \{e_1, e_2\}$, $E' = \{e'_1, e'_2\}$. Also BPNS sets are

$$A_1 = \left\{ \langle e_1, \{(u_1, 0.4, 0.3, 0.1, -0.4, -0.4, -0.7), (u_2, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4)\} \rangle, \right. \\ \left. \langle e_2, \{(u_1, 0.4, 0.5, 0.2, -0.1, -0.2, -0.4), (u_2, 0.6, 0.2, 0.1, -0.1, -0.4, -0.4)\} \rangle \right\}$$

$$A_2 = \left\{ \langle e_1, \{(u_1, 0.5, 0.2, 0.1, -0.6, -0.2, -0.4), (u_2, 0.6, 0.3, 0.1, -0.6, -0.1, -0.1)\} \rangle, \right. \\ \left. \langle e_2, \{(u_1, 0.5, 0.2, 0.2, -0.3, -0.1, -0.2), (u_2, 0.7, 0.1, 0.1, -0.3, -0.1, -0.3)\} \rangle \right\}$$

$$A_3 = \left\{ \langle e'_1, \{(v_1, 0.3, 0.5, 0.4, -0.2, -0.5, -0.8), (v_2, 0.1, 0.7, 0.4, -0.3, -0.3, -0.5)\} \rangle, \right. \\ \left. \langle e'_2, \{(v_1, 0.2, 0.6, 0.4, -0.1, -0.4, -0.4), (v_2, 0.4, 0.3, 0.2, -0.1, -0.5, -0.5)\} \rangle \right\}$$

Then, $\tau = \{0, 1, A_1, A_2\}$ and $\sigma = \{0, 1, A_3\}$ are BPNST on U and V respectively. Now define a mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ such that $f(u_1) = v_1$, $f(u_2) = v_2$ and $f(e) = e'$. Therefore f is BPNSGP-continuous mapping.

Theorem 4.3 Let (U, τ, E) and (V, σ, E') be a any two bipolar pythagorean neutrosophic soft topological spaces. Then,

1. Every BPNS-continuous mapping is a BPNSGP-continuous mapping.
2. Every BPNSG-continuous mapping is a BPNSGP-continuous mapping.
3. Every BPNSP-continuous mapping is a BPNSGP-continuous mapping.
4. Every $BPNS\alpha$ -continuous mapping is a BPNSGP-continuous mapping.
5. Every $BPNSG\alpha$ -continuous mapping is a BPNSGP-continuous mapping.
6. Every BPNSGP -continuous mapping is a BPNSSP-continuous mapping.
7. Every BPNSGP -continuous mapping is a BPNSGSP-continuous mapping.

Proof: 1. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNS-continuous mapping. Suppose A is BPNSCS in V , then $f^{-1}(A)$ is BPNSCS in U . Since every BPNSCS is BPNSGPCS, $f^{-1}(A)$ is BPNSGPCS in U . Therefore f is BPNSGP- continuous mapping.

2. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSG-continuous mapping. Suppose A is BPNSGCS in V , then $f^{-1}(A)$ is BPNSGCS in U . Since every BPNSGCS is BPNSGPCS, $f^{-1}(A)$ is BPNSGPCS in U . Therefore f is BPNSGP- continuous mapping.

3. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSP-continuous mapping. Suppose A is BPNSPCS in V , then $f^{-1}(A)$ is BPNSPCS in U . Since every BPNSPCS is BPNSGPCS, $f^{-1}(A)$ is BPNSGPCS in U . Therefore f is BPNSGP- continuous mapping.
4. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNS α -continuous mapping. Suppose A is BPNS α CS in V , then $f^{-1}(A)$ is BPNS α CS in U . Since every BPNS α CS is BPNSGPCS, $f^{-1}(A)$ is BPNSGPCS in U . Therefore f is BPNSGP- continuous mapping.
5. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSG α -continuous mapping. Suppose A is BPNSG α CS in V , then $f^{-1}(A)$ is BPNSG α CS in U . Since BPNSG α CS is BPNSGPCS, $f^{-1}(A)$ is BPNSGPCS in U . Therefore f is BPNSGP- continuous mapping.
6. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSGP-continuous mapping. Suppose A is BPNSGPCS in V , then $f^{-1}(A)$ is BPNSGPCS in U . Since BPNSGPCS is BPNSSPCS, $f^{-1}(A)$ is BPNSSPCS in U . Therefore f is BPNSSP- continuous mapping.
7. Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSGP-continuous mapping. Suppose A is BPNSGPCS in V , then $f^{-1}(A)$ is BPNSGPCS in U . Since BPNSGPCS is BPNSGSPCS, $f^{-1}(A)$ is BPNSGSPCS in U . Therefore f is BPNSGSP- continuous mapping.

Remark: 4.4 Converse of theorem 4.3 is not true.

Example: 4.5 1. Let $U = \{u_1, u_2, u_3\}$, $V = \{v_1, v_2, v_3\}$ and $E = \{e_1, e_2\}$, $E' = \{e'_1, e'_2\}$. Also BPNS sets are

$$A_1 = \left\{ \left\langle e_1, \left\{ (u_1, 0.4, 0.3, 0.1, -0.4, -0.4, -0.7), (u_2, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle, \right. \\ \left. \left\langle e_2, \left\{ (u_1, 0.4, 0.5, 0.2, -0.1, -0.2, -0.4), (u_2, 0.6, 0.2, 0.1, -0.1, -0.4, -0.4), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle \right\}$$

$$A_2 = \left\{ \left\langle e_1, \left\{ (u_1, 0.5, 0.2, 0.1, -0.6, -0.2, -0.4), (u_2, 0.6, 0.3, 0.1, -0.6, -0.1, -0.1), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle, \right. \\ \left. \left\langle e_2, \left\{ (u_1, 0.5, 0.2, 0.2, -0.3, -0.1, -0.2), (u_2, 0.7, 0.1, 0.1, -0.3, -0.1, -0.3), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle \right\}$$

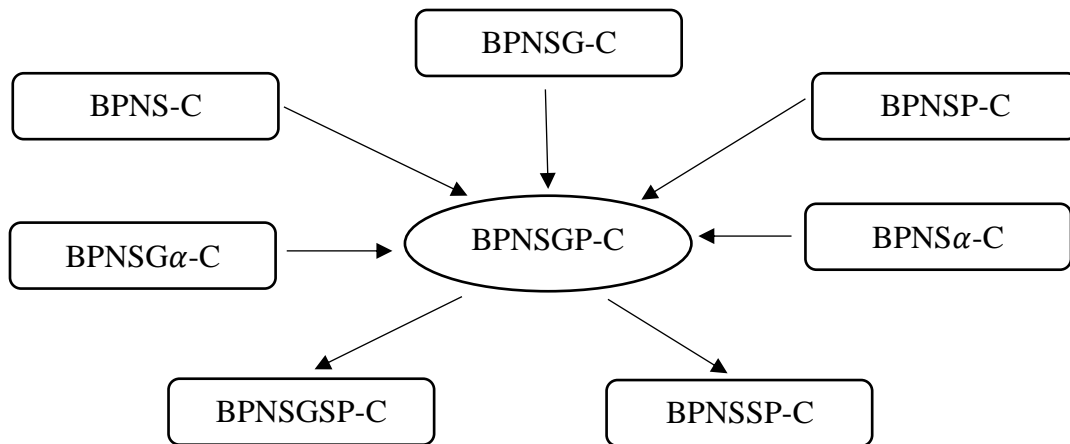
$$A_3 = \left\{ \left\langle e'_1, \left\{ (v_1, 0.3, 0.5, 0.4, -0.2, -0.5, -0.8), (v_2, 0.1, 0.7, 0.4, -0.3, -0.3, -0.5), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle, \right. \\ \left. \left\langle e'_2, \left\{ (v_1, 0.2, 0.6, 0.4, -0.1, -0.4, -0.4), (v_2, 0.4, 0.3, 0.2, -0.1, -0.5, -0.5), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4) \right\} \right\rangle \right\}$$

Then, $\tau = \{0, 1, A_1, A_2\}$ and $\sigma = \{0, 1, A_3\}$ are BNST on U and V respectively. Now define a mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ such that $f(u_1) = v_1$, $f(u_2) = v_2$ and $f(e) = e'$. Therefore f is BPNSGP-continuous mapping but not BPNS- continuous mapping. Because,

$$A_3^c = \left\{ \left\langle e'_1, \left\{ (v_1, 0.4, 0.5, 0.3, -0.8, -0.5, -0.2), (v_2, 0.4, 0.3, 0.1, -0.5, -0.7, -0.3), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.3, 0.4, 0.2, -0.4, -0.8, -0.5) \right\} \right\rangle, \right. \\ \left. \left\langle e'_2, \left\{ (v_1, 0.4, 0.4, 0.2, -0.4, -0.6, -0.1), (v_2, 0.2, 0.7, 0.4, -0.5, -0.5, -0.1), \right. \right. \right. \\ \left. \left. \left. (u_3, 0.3, 0.4, 0.2, -0.4, -0.8, -0.5) \right\} \right\rangle \right\}$$

is BPNSCS in V , but $f^{-1}(A_3^c)$ is not BPNSCS in U .

The remaining examples are similar.



The relation between various types of BPNS continuous mappings

Theorem: 4.6 A mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ is BPNSGP-continuous if and only if the inverse image of each bipolar pythagorean neutrosophic soft open set (BPNSOS) in V is bipolar pythagorean neutrosophic soft generalized pre-open set (BPNSGPOS) in U .

Proof: Consider a BPNSOS A in V . Then its complement is BPNSCS in V . Since f is BPNSGP-continuous mapping, $f^{-1}(A^c)$ is BPNSGPCS in U . Also, since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is BPNSGPOS in U . By the definition of BPNSGP-continuous mapping the converse part is straight forward.

Theorem: 4.7 Consider a BPNSGP-continuous mapping and bipolar pythagorean neutrosophic soft continuous mapping $f_1: (U, \tau, E) \rightarrow (V, \sigma, E')$ and $f_2: (U, \sigma, E') \rightarrow (W, \gamma, E'')$ respectively. Then $f_1 \circ f_2: (U, \tau, E) \rightarrow (W, \gamma, E'')$ is a BPNSGP-continuous mapping.

Proof: Assume that A is BPNSCS in W . Then, by hypothesis $f_2^{-1}(A)$ is BPNSCS. Since f_1 is BPNSGP-continuous mapping, $f_1^{-1}(f_2^{-1}(A))$ is BPNSGPCS in U . Hence $f_1 \circ f_2$ is a BPNSGP-continuous mapping.

Definition: 4.8 If (U, τ, E) is a BPNST. Then the bipolar pythagorean neutrosophic soft generalized pre-interior (BPNSgpint) and bipolar pythagorean neutrosophic soft generalized pre-closure (BPNSgpcl) for any BPNSS A is defined by,

$$\text{BPNSgpint}(A) = \cup \{P | P \text{ is a BPNSGPOS in } U \text{ and } P \subseteq A\}$$

$$\text{BPNSgpcl}(A) = \cap \{Q | Q \text{ is a BPNSGPCS in } U \text{ and } A \subseteq Q\}$$

If A is BPNSGPCS, then $\text{BPNSgpcl}(A) = A$.

Theorem: 4.9 Consider a mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ as BPNSGP-continuous mapping. Then:

i) For every BPNSS A in U , $f(\text{BPNSgpint}(A)) \subseteq \text{BPNScl}(f(A))$.

ii) For every BPNSS A in U , $\text{BPNSgpcl}(f^{-1}(A)) \subseteq f^{-1}(\text{BPNScl}(A))$.

Proof: i) As $\text{BPNScl}(f(A))$ is BPNSCS in V and f is BPNSGP-continuous mapping, then $f^{-1}(\text{BPNScl}(f(A)))$ is BPNSGPCS in U . So, $\text{BPNSgpcl}(A) \subseteq f^{-1}(\text{BPNScl}(f(A)))$. Therefore $f(\text{BPNSgpint}(A)) \subseteq \text{BPNScl}(f(A))$ for every BPNSS A in U .

ii) Replace A by $f^{-1}(A)$ in (i), then we have $f(\text{BPNSgpcl}(f^{-1}(A))) \subseteq \text{BPNScl}(f(f^{-1}(A))) \subseteq \text{BPNScl}(A)$. Therefore $\text{BPNSgpcl}(f^{-1}(A)) \subseteq f^{-1}(\text{BPNScl}(A))$, for every BPNSS A in U .

5 Bipolar Pythagorean Neutrosophic Soft Generalized Pre-Irresolute Mappings

Definition: 5.1 If $f^{-1}(A)$ is BPNSGPCS in (U, τ, E) for every bipolar pythagorean neutrosophic soft (BPNS) generalized pre closed set A of (V, σ, E') then $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ is called as bipolar pythagorean neutrosophic soft generalized pre-irresolute (BPNSGP-irresolute) mapping.

Example: 5.2 Let $U = \{u_1, u_2\}$, $V = \{v_1, v_2\}$ and $E = \{e_1, e_2\}$, $E' = \{e'_1, e'_2\}$. Also BPNS sets are

$$A_1 = \left\{ \langle e_1, \{(u_1, 0.4, 0.3, 0.1, -0.4, -0.4, -0.7), (u_2, 0.2, 0.6, 0.3, -0.5, -0.2, -0.4)\} \rangle, \right. \\ \left. \langle e_2, \{(u_1, 0.4, 0.5, 0.2, -0.1, -0.2, -0.4), (u_2, 0.6, 0.2, 0.1, -0.1, -0.4, -0.4)\} \rangle \right\}$$

$$A_2 = \left\{ \langle e_1, \{(u_1, 0.5, 0.2, 0.1, -0.6, -0.2, -0.4), (u_2, 0.6, 0.3, 0.1, -0.6, -0.1, -0.1)\} \rangle, \right. \\ \left. \langle e_2, \{(u_1, 0.5, 0.2, 0.2, -0.3, -0.1, -0.2), (u_2, 0.7, 0.1, 0.1, -0.3, -0.1, -0.3)\} \rangle \right\}$$

$$A_3 = \left\{ \langle e'_1, \{(v_1, 0.3, 0.5, 0.4, -0.2, -0.5, -0.8), (v_2, 0.1, 0.7, 0.4, -0.3, -0.3, -0.5)\} \rangle, \right. \\ \left. \langle e'_2, \{(v_1, 0.2, 0.6, 0.4, -0.1, -0.4, -0.4), (v_2, 0.4, 0.3, 0.2, -0.1, -0.5, -0.5)\} \rangle \right\}$$

Then, $\tau = \{0,1,A_1,A_2\}$ and $\sigma = \{0,1,A_3\}$ are BPNST on U and V respectively. Now define a mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ such that $f(u_1) = v_1$, $f(u_2) = v_2$ and $f(e) = e'$. Therefore f is BPNSGP-irresolute mapping.

Theorem: 5.3 Consider a BPNSGP-irresolute mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$, then f is BPNSGP-continuous mapping but its converse is not true.

Proof: Assume that f is BPNSGP-irresolute mapping and assume a BPNSCS A in V . Every BPNSCS is BPNSGPCS, then A is BPNSGPCS in V . Also, f is BPNSGP-irresolute mapping. Thus, by definition $f^{-1}(A)$ is BPNSGPCS in U . So, f is BPNSGP-continuous mapping.

Theorem: 5.4 Let $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ be BPNSGP-irresolute mapping if and only if the inverse of every BPNSGPOS in V is BPNSGPOS in U .

Proof: Consider a BPNSGPOS A in V . Then its complement is BPNSGPCS in V . Since f is BPNSGP-irresolute mapping, $f^{-1}(A^c)$ is BPNSGPCS in U . Thus $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is BPNSGPOS in U . The converse part is straight forward.

Theorem: 5.5 Consider $f_1: (U, \tau, E) \rightarrow (V, \sigma, E')$ and $f_2: (U, \sigma, E') \rightarrow (W, \gamma, E'')$ as BPNSGP-irresolute mapping, where U, V, W are BPNSGTS, then $f_1 \circ f_2$ is BPNSGP-irresolute mapping.

Proof: Assume BPNSGPCS A in W . Since f_1 is BPNSGP-irresolute mapping, $f_2^{-1}(A)$ is BPNSGPCS in V . Since f_1 is BPNSGP-irresolute mapping, $f_1^{-1}(f_2^{-1}(A))$ is BPNSGPCS in U . Thus $(f_1 \circ f_2)^{-1}(A)$ is BPNSGPCS in U . Hence $f_1 \circ f_2$ is a BPNSGP-irresolute mapping.

6 Conclusion

In this paper, BPNS generalized pre-continuous and irresolute mappings are introduced. It shows how the topological structures such as closeness and openness are preserved by different types of continuities. In future, application of the developed mapping will be studied.

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